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**MASTER’S THESIS**

**Topic:** Интерферометрические измерения оптических вихрей  
(Interferometric measurement of optical vortices)

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TASK FOR THE MASTER’S THESIS

Approved
Head of the department LINS
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June 2018

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Topic: Интерферометрические измерения оптических вихрей (Interferometric measurement of optical vortices)

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The research on properties of optical vortices began with the article of D. P. Ghai, R. S. Sirohi and P. Senthilkumaran that was published under the name “Shero-grams of an optical phase singularity” Opt. Com. A281, 1315-1322 in (2008).

Contents of the thesis:
Briefly illustrate the main properties of optical vortex and its detection using shearing interferometers.

List of report materials: the text of the GQW, illustrations, other reporting materials

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# PROJECT TIMELINE FOR THE MASTER’S THESIS

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Student: Али Мермуль (Ali MERMOUL)  
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SUMMARY

Explanatory note 83 p., 53 fig., 00 charts, 43 sources, 03 tables, 01 appendix.

The subject of the research (development) is: Interferometric measurement of optical vortices.

The target of the GQW – The detection of orbital angular momentum in the optical vortices using rotational and reversal shearing interferometries. The dissertation provides a description about the detection of orbital angular momentum of optical vortices using shearing interferometries, the optical vortices were generated using computer generated hologram. The rotational and reversal shearing interferometers where used for the detection. The dissertation also discusses introduction and some principles of Holography; In addition to that, Optical vortices and the generation methods were discussed briefly. Then we give a general view about interferometry, its types and how to measure orbital angular momentum of light. Finally, the experiment part that consists of a description of design, experiment of optical vortices generation, then a description of both rotational and reversal shearing interferometry setups used for detection of optical vortices orbital angular momentum. In the end, comments and explanations about the results were given.
В диссертации дано описание обнаружения орбитального углового момента оптических вихрей с помощью сдвиговых интерферометров, оптические вихри были сгенерированы с помощью компьютерной голограммы. Интерферометры вращательных и реверсирования режа где использованный для обнаружения. В диссертации также обсуждается введение и некоторые принципы Голографии, кроме того, были кратко обсуждены оптические вихри и методы генерации. Затем мы даем общее представление об интерферометрии и ее типах и о том, как измерять орбитальный угловой момент света. Наконец, на экспериментальной части, состоящей из описания конструкции, эксперимента генерации оптических вихрей, затем описания как вращательных, так и реверсивных сдвиговых интерферометрических установок, используемых для обнаружения орбитальных угловых моментов оптических вихрей. В итоге были даны комментарии и разъяснения по поводу полученных результатов.
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DEFINITIONS, DESIGNATIONS AND ABBREVIATIONS

The present explanatory note uses the following abbreviations and designations:

2D – Two dimensional
3D – Three dimensional
AM – Amplitude modulation
AOR – Artificial optical radiation
BSI – British standards institution
CBS – Cubic beam splitter
CCW – Counter clockwise
CGH – Computer generated hologram
CoSHH – Control of substances hazardous to health
CVB – Cylindrical vector beams
CW – Clockwise
DEHI – Double exposure holographic interferometer
DM – Deformable mirror
Dr. – Doctor in science
FH – Fork hologram
He-Ne – Helium-Neon
HG – Hermite-Gauss
HI – Holographic interferometer
HOE – Holographic optical element
IEC – International electrotechnical commission
IR – Infrared radiation
LADAR – Laser detection and ranging
LCD – Liquid crystal display
LG – Laguerre-Gauss
LASER – Light amplification by stimulated emission of radiation
LSI – Lateral shearing interferometer
MI – Michelson interferometer
OAM – Orbital angular momentum
OV – Optical vortex
OPD – Optical path difference
PC – Personal computer
PM – Phase modulation
PSHI – Phase shifting holographic interferometer
QKD – Quantum key distribution
RTHI – Real time holographic interferometer
RtSI – Rotational shearing interferometer
RvSI – Reversal shearing interferometer
SDM – Segmented deformable mirror
SI – Shearing interferometer
SLM – Spatial light modulator
SOP – State of Polarization
TC – Topological charge
VRtSI – Variable rotational shearing interferometer
INTRODUCTION

In the last century there have been a lot of discoveries such as discovery of neutrons, DNA helical structure, black holes, universe expansion…etc and many inventions such as radar, airplanes, television, the internet, electron microscope …etc which become the foundation of the modern technology. Furthermore, there are two extraordinary inventions that shaped the world to become to as we know it today which are the laser and the transistor. Before the invention of laser, the holography invention didn’t take a lot of attention by the scientists because it needed a coherent light to be functional. However, after 1960, when Theodore H. Maiman built his ruby laser which become the first laser ever, the holography became of a great interest, since it can visualize the objects as a 3D images called holograms, which lead to myriad number of applications in security, art, interferometry, microscopy, HOE…etc[1,2]. Furthermore, holography is used as a basic foundation for solving the information paradox of black holes via holographic principle.

In the 1974, another important discovery in optical science field was made, when Berry and Nye on their research on the reflections of radio waves off of the Antarctic ice sheets, when they noticed that some regions have a unique properties where the signal intensity was nil at midpoint and had $2\pi$ phase change, which in fact was vortices generated from ultrasound waves interference off of the rough surface [3]. Nowadays, it was proven that when three or more plane waves interfere with each other, the resultant pattern always contain many vortices [4, 5].

After this discovery, a new branch of modern optics was born which was named as Singular optics by Prof. Marat Soskin. The singular optics field is divided to three groups: phase singularities, ray singularities and polarization singularities [6]. Where the phase singularities are points with zero intensity and the phase is unknown.

Since 1950’s it was confirmed that transitions with higher order may hold several units of ħ, which means it has spin angular momentum plus orbital angular momentum [7].
A lot of attention was driven to optical vortices by scientists and researches since they have unique characteristics such as carrying orbital angular momentum and the helical shape. Because of the infinite degrees of freedom that an orbital angular momentum has, optical vortices found their way too many fields and applications such as like processes of quantum information, optical tweezers and multi-level quantum key distribution (QKD) [8-11].

The topic in this project is to study the generation of optical vortices using computer generated hologram (CGH) and when they interfere with themselves by using shear interferometers, especially rotational and reversal shearing interferometers.

Chapter 1 presents two subjects by starting first with the holography history, its fundamentals and classification, since our vortex generation is based on CGH. The second subject is optical vortices, where we will discuss the properties and generation of optical vortices.

Chapter 2 discusses interferometry fundamentals, shearing interferometry and shearograms.

Chapter 3 represents the experimental setups for generation of optical vortices, optical vortices with shearing interferometry setups and the results of the experiments.

Chapter 4 discusses the safety procedures.
1. HOLOGRAPHY AND OPTICAL VORTEX

1.1. Holography

Holography is an extraordinary invention that revolutionized the world of imaging techniques. The holography which originally a Greek word, also called lensless photography. In photography, the image is 2D representation of a three dimensional view even if the optical elements used to for recording generate a three dimensional images, as we may know the telescope basically generate a three dimensional pictures. This is because in photography only the information guarding the amplitude of light waves whereas the phase information is missing from the recording medium, or in other words the photography is 2D registration of a 3D scene only by amplitude information, where the depth perception or parallax is absent and all the information on the relative phases of light waves from the original three dimensional view is lost, we can no longer modify the perspective of the photography image by viewing it from different angles (Parallax).

Now, the only method to save the phase of light is by applying interference phenomena. The holography uses both the interference and diffraction to produce a 3D image, where the recorded hologram has all the needed image field information to create a 3D image and it provides depth perception and parallax. Unlike the photography, the hologram preserves the original object three dimensionality.

In holography, what is saved in the recording medium is the interference pattern produced by the two interfering waves and the intensity at any point in the interferometric pattern depends on the amplitude plus the phase of the original object beam. The ability of holography to record the intensity plus the phase of the object wave is the main feature that differentiate the holography from photography.

It seems incredible that to be able to record into a two dimensional recording medium all the information of a three dimensional object.

1.1.1. History of holography

The story of holography begins at 1947, when the Hungarian researcher, scientist and physicist was working for the improvement of electron microscope,
when he proposed a technique of mixing the wave containing the needed information with a second plane wave to form a coded intensity distribution on the photographic plate to overcome the deficiency of phase in photography. The nature of holography originally proposed by Gabor, utilizes an inline setup (figure 1), it was a straightforward holography technique that places the object and the light source on the axis perpendicular to the holographic plate where a small semi-transparent object is lightened by a coherent light source. The light scattered from the object creates a second wave that superposes with the reference beam originating from the light source on the photographic plate.

![Figure 1 - The in-line recording and reconstruction setups](image)

Gabor indicated that without the use of lenses, the original image-forming wave could be reconstructed from the hologram to provide a representation of the original object; the only catch here is the procedure depends on using sufficiently coherent light for the beams to be in phase in their path which wasn’t available at that time.

The Gabor original scheme has many disadvantages such that it needed only semi-transparent object. After the reconstruction of the hologram and during observation an image disturbances created when the two images (virtual and real image) lying on the same axis interfere, this is infamously recognized as the twin-image problem. Moreover, the observer looks straight into the reconstruction wave. It is called the “single beam holography” because a single laser beam is used in recording without splitting it.

In the 1962 the first transmission holography was invented by Leith and Upatnieks from Michigan University, where their holography technique overcome the twin-image problem, what they did is that they tilted the reference beam or shift the ob-
ject to become off-axis so the three diffraction orders: the illumination wave, the image and the conjugated image are spatially separated which means we avoid the undesired overlapping of the real and virtual image suffered by the inline recording method. Also, the opaque object holograms can be created.

![Image of Off-axis recording setup]

Figure 2 - The Off-axis recording setup

In Russia in the 1962, the first reflection holography technique came to the world, when Dr. Yuri Denisyuk joined Lippmann’s effort in natural color photography to create a white-light reflection hologram. It was for the first time a hologram can be viewed using ordinary light bulb or sun light.

![Image of Dr. Yuri Denisyuk with his reflection hologram]

Figure 3 - Dr. Yuri. Denisyuk with his reflection hologram

1.1.2. Holography generation fundamentals

The holography is a method for registering all the information of the scattered light from an object, we mean by information the amplitude and phase of the light beam and later reconstruct it when the original light waves are no longer available due to the original object absence. The holography can be known as the second order photography, where the holography recording and reconstruction procedures are very similar to the writing and reading process of a CD and whereas photography only records the square of the electric field, a holograms holds both
phase and amplitude information from the image which explains the requirement for high-power illumination. Since more information is being stored in the hologram recording medium than in the photographic film, it should be logically understood that a hologram plate should need more light energy than a conventional 2D photograph.

To obtain a 3D image using holography, first few equipments and requirements must be obtained to create a holographic system such as test object, laser beams and recording medium.

1.1.2.1. Beam source

One of the differences that differentiate the holography from the photography is that the holography always needs a coherent light for recording process, unlike the usual light sources or even the sun light which are incoherent and contain myriad wavelengths.

The laser in another hand, the Laser is the best choice for recording a 3D image. The field depth which could be registered in the plate is determined by the laser’s coherence length. So to obtain a good hologram the laser’s coherence length should be more than few meters. For holography process, the light source requirements are: Monochromaticity, Coherence, stable output, High intensity and power. Nowadays, the most used lasers in the holography process are He-Ne lasers, because they are stable have a high intensity and large coherence length.

![Figure 4 - He-Ne laser](image)

1.1.2.2. Recording medium

To attain a good hologram, a film with high enough resolution to register the interference pattern created by the object and reference beam is required. The conversion that the original interference patterns into an optical element that changes either the incident light beam phase or amplitude in proportion to the in-
tensity of original light beam is the main job of the recording film. Usually, the
recording plate (Holoplate) is similar to silver Halide photographic emulsion. How-
ever, it has much higher light-sensitive grains concentration to completely resolve
the fringes created from the interfering reference and object beams.
Development of the recording pattern is a critical matter, which converts exposed
silver bromide into metallic silver, or in other words, it forms an invisible (latent)
image into visible image. Where the table 1 below shows some of the recording
media used in holography.

![Figure 5 - Silver halide film illustration](image)

Table 1 - Overview of the recording media in holography

<table>
<thead>
<tr>
<th>Material</th>
<th>Process</th>
<th>Hologram type</th>
<th>Exposure (\text{mJ/cm}^2)</th>
<th>Spectrum (\mu\text{m})</th>
<th>Diffir. eff.</th>
<th>Resolution (lines/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not erasable silver halide</td>
<td>Developing</td>
<td>Amplitude</td>
<td>0.001-0.1</td>
<td>0.4-0.7</td>
<td>0.05</td>
<td>3-7</td>
</tr>
<tr>
<td>Erasable Photochr.</td>
<td>None</td>
<td>Amplitude</td>
<td>10-100</td>
<td>0.3-0.7</td>
<td>0.02</td>
<td>&gt;5</td>
</tr>
<tr>
<td>Thermopl.</td>
<td>Ch., heat</td>
<td>Phase</td>
<td>0.01</td>
<td>0.4-0.65</td>
<td>0.3</td>
<td>0.5-1.2</td>
</tr>
<tr>
<td>Photorefr.</td>
<td>None</td>
<td>Phase</td>
<td>10-100</td>
<td>0.35-0.5</td>
<td>0.2</td>
<td>&gt;1.5</td>
</tr>
<tr>
<td>“+Photol.”</td>
<td>None</td>
<td>Phase</td>
<td>0.1</td>
<td>0.35-0.5</td>
<td>0.25</td>
<td>&gt;10</td>
</tr>
<tr>
<td>Silver halide</td>
<td>Bleaching</td>
<td>Phase</td>
<td>0.001-0.1</td>
<td>0.4-0.7</td>
<td>≥ 0.5</td>
<td>3-7</td>
</tr>
<tr>
<td>Dichrom.</td>
<td>Developing</td>
<td>Phase</td>
<td>10</td>
<td>0.35-0.58</td>
<td>0.9</td>
<td>&gt;10</td>
</tr>
<tr>
<td>Photoresist</td>
<td>Developing</td>
<td>Phase</td>
<td>10</td>
<td>UV-0.5</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>Photopol.</td>
<td>Exposure</td>
<td>Phase</td>
<td>1-1000</td>
<td>UV-0.65</td>
<td>0.9</td>
<td>0.2-1.5</td>
</tr>
</tbody>
</table>
1.1.2.3. Further apparatus

For most holography system structures, the beam coming out from the laser needed to be divided to two beams: the reference beam and the object beam, while the former goes directly to the recording plate, the latter hit the object and the scattered rays, falls onto the emulsion. However, it is nearly impossible to do that without using beam splitters (Fig. 6), mirrors (Fig. 7) and lenses (Fig. 8) are used to expand the object light to the object. And for creating a clear hologram, a stable table is needed.

Furthermore, the most important thing in the whole holographic procedure is to provide a (static or moving) object (Fig. 9).

1.1.3. The theory of holography

1.1.3.1. Holographic recording procedure

To ensure that the hologram will be perfect, it is recommended to use a dark room where the external light has an insignificant intensity and to create holograms and 3D images, tools like lenses, beam splitters, holoplates, laser, mirrors, steady table and of course the test object that we want to be recorded are necessary. First
the laser light is directed toward the object, but before it hit it, it passes through a beam splitter, where two coherent and identical light waves are achieved, after that they pass through lenses to stretch upon the whole test object and the whole holo-plate. Yet, before that mirrors are used to direct beams to right path. The two beams that are obtained by the beam splitter are called the reference beam and the object beam. The former one is directed by mirrors toward the recording plate, however, it should not interfere with any other light or object in the way to the recording medium. The latter one is expended by the lenses and directed by mirror to hit the test object, off course some of the light will be absorbed and scattered, the scattered and reflected beams find their way to the recording plate. Where the interference shape imprinted on the holo Plate is the result of the interaction and the interference of the two light beams. The figure 10 shows one of the myriad procedures to record a hologram.

![Hologram recording process](image)

When an interference of the two light beams happen, the process of recording a hologram start, the resulting imprinted interference pattern has fringe distances of few µm, which is in the order of wavelength of the laser light. The information of the object beam is hidden in the distance between fringes and fringes intensity modulation (AM or PM).
After processing, the recording medium seems to be featureless and hold no resemblance to the object. Yet, the interference detail on the plate becomes evident when the holoplate is enlarged.

Now, because in every system there is always a background noise, that is way the hologram light is never as good as the light scattered from the object.

The intermodulation noise can be minimized by ensuring that the reference light has a higher intensity than the light coming from any point on the object.

The electric or magnetic field of light wave can be modeled by complex number $U$, so the angle of $U$ is phase of light and the amplitude of light beam is the magnitude of $U$. $U_{\text{obj}}$ and $U_{\text{ref}}$ are the object and reference beams at any place in the holographic arrangement, respectively. Now, in the recording plate, $U_{\text{obj}} + U_{\text{ref}}$ is the result of the combined beams, the square of the magnitude of the electric or magnetic field of the combined beam is proportional to the energy $I$.

$$I = \left| U_{\text{ref}} + U_{\text{obj}} \right|^2 = U_{\text{obj}}^* U_{\text{ref}} + \left| U_{\text{ref}} \right|^2 + \left| U_{\text{obj}} \right|^2 + U_{\text{ref}} U_{\text{obj}}^*.$$  \hfill (1)

1.1.3.2. Holographic reconstruction process

The procedure of reconstruction comes after the plate development, first the hologram is illuminated by a laser light identical to the reference beam used in the recording process, this leads to an exact generation of the original object beam. A camera or an eye will see exactly the same view as it would have done when observing the original object. The image changes when changing the angle of observation the same way as it would have done when the test object was in the same place of recording process.
When the two beam are exposed to the recording medium, then developed, the photographic plate has transmittance $T$ proportional to the energy of the incident light on the plate, it is given by:

$$T = C \left[U_{obj} U_{ref}^* + |U_{ref}|^2 + |U_{obj}|^2 + U_{ref} U_{obj}^* \right].$$  \hfill (2)

Now, $C$ is a constant, after development and in the reconstruction process when the reference beam illuminate the developed holoplate $U_{tra}$ is the light transmitted through the plate is:

$$U_{tra} = T U_{ref} = C \left[U_{obj} U_{ref}^* + |U_{ref}|^2 + |U_{obj}|^2 + U_{ref} U_{obj}^* \right] U_{ref};$$  \hfill (3)

$$U_{tra} = C \left[U_{ref}^2 U_{obj}^* + U_{ref} |U_{ref}|^2 + U_{ref} |U_{obj}|^2 + U_{obj} \right].$$

The equation (3) shows that there are four terms, the first one is identified as the “the conjugate object light”, forming the real image of the object and causing the reverse curvature of the object light are its properties. The second terms and third terms denotes the modified amplitude of the reference beam by $|U_{ref}|^2$ and $|U_{obj}|^2$, respectively. The last terms which is $U_{obj}$ is the re-constructed object light.

### 1.1.4. Holography taxonomy

The classification of holograms may depend on criteria such as: usage and the study area, the used optic and electronic elements, the holographic object
(moving or static), the reference and object beam, copy and multiplication (broken holograms, hologram’s hologram), illumination methods of recording, construction stages (H1, H2, H3…), Dimensions (thick and thin holograms, Small and large holograms).

Where holography types are concluded in the following table:

<table>
<thead>
<tr>
<th>Table 2 - Holography types</th>
</tr>
</thead>
<tbody>
<tr>
<td>Types</td>
</tr>
<tr>
<td>----------------------------</td>
</tr>
<tr>
<td>In-line/off-axis</td>
</tr>
<tr>
<td>Thick/thin</td>
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<tr>
<td>Amplitude/phase</td>
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<tr>
<td>Transmission/reflection</td>
</tr>
<tr>
<td>Fresnel/Fraunhofer/Fourier</td>
</tr>
<tr>
<td>Laser/ white light</td>
</tr>
</tbody>
</table>

1.1.4.1. **Amplitude hologram and Phase hologram**

The recording of both the amplitude and phase of the electric field in the same material is impossible, that is why there is basically two kinds of holographic processes, the amplitude modulation and the phase modulation.

In the phase modulation, it is made when the refractive index or the thickness of the material are changed in proportion to the holographic interference pattern intensity, the energy in the hologram is redirected to the wanted region/image, it is required that the holograms to be totally dried before viewing in the PM Holograms, because during the developing procedure the emulsion swelling distorts the surface which lead to damaging the hologram.
In the amplitude modulation, transmission variation is the registration of the phase information and the intensity of the recorded light is proportional to the amplitude of beams diffracted by the hologram. The amplitude modulation has a significant amount of light loss since it modulate the light mainly through absorption which means low diffraction efficiency compared to the phase modulation. Because the light is not blocked by dark fringes in phase modulation holograms, they are more efficient.

1.1.4.2. Reflection versus transmission holograms

The reflection hologram and transmission hologram main difference lies in the interference fringes direction that are registered inside the recording emulsion. Whereas in the transmission hologram, the two interfering beams are entering the emulsion from the same side, where the plane of the emulsion is perpendicular to the produced interference fringes planes. The transmission holograms have a remarkable properties, the first is that the information from all original object points are included in the entire hologram, that means the information to reconstruct the whole hologram is contained in any part of the hologram, which means that every piece of the broken transmission hologram will give a the full image the original object. Second, we can observe the behind and around the objects just by repositioning the hologram, the depth of field is present.

Now, for the reflection hologram, the interfering lights are entering the recording plate from different sides, so the plane of the plate is parallel to the produced interference fringes planes. As seen near its surface, the reflection hologram is a truly three-dimensional images. The big advantage of reflection hologram is that it doesn’t need the same recording laser beam to reconstruct the hologram, due to Bragg diffraction, only the used wavelength for the recording is reflected if a white light is used for illumination. However the field depth of transmission hologram is greater reflection holograms.
1.2. Optical vortex

In the 1970, another important discovery in optical science field was made, when Berry and Nye on their research on the reflections of radio waves off of the Antarctic ice sheets, when they noticed that some regions have a unique properties where the signal intensity was nil and $2\pi$ phase change, which in fact was vortices generated from ultrasound waves interference off of the rough surface [3]. Nowadays, it was proven that when three or more plane waves interfere with each other, the resultant pattern always contain many vortices [4, 5]. Since 1950’s it was confirmed that transitions with higher order may hold several units of $\hbar$, which means it has spin angular momentum plus orbital angular momentum [7].
Nowadays, optical vortices are used in many applications around the world such as particle trapping, LADAR (laser detection and ranging) systems, nanotechnologies, biology, optical communication field, laser beam shaping, optical tweezers and coronagraphs of optical vortices [8,12-15].

1.2.1. Overview of optical vortex

In a wavefront, an isolated phase singularity is called an optical vortex which has phase dislocation of a screw type. The amplitudes of the complex field real part and the imaginary part are equal to zero because at the singular point the phase is unspecified. An electromagnetic wave fields that have a structure with a peculiar phase are called screw dislocation or phase singularities (in quantum physics realm are called photonic quantum vortices). These screw dislocation waves have a helicoidal wavefronts and own phase singularities, during propagation through a disturbing media, they present properties such as self-healing and great robustness which make them a special waves. OV is a continuous surface that has an embedded $l$ helicoids, where each one has $\lambda$ pitch, and separated with wavelength $\lambda$. The figure 14 shows ideal structure, phase and intensity of an optical vortices.

An optical vortex which are also famous as Laguerre-Gaussian tweezer [16] spiral phase wavefront rotating around its optical axis which cause corkscrew twisting during its propagation. An unlimited amount of OVs with alternating charges are generated from the line of phase singularity (half-cut), however, these OVs move away from the beam’s observable part slowly and mod(m) OV are only remained charges in the far field.

The $\varphi (\rho, \theta) = l \theta$ is the phase singularity’s transverse optical phase in the cylindrical coordinates. In the classical denotation, the topological charge is denoted as “$l$” where $-\infty < l < \infty$, which is the identical quantized OAM denotation.

Moreover, mathematically, an isolated point that fulfills the succeeding line integral is always a vortex:

$$\oint \nabla \varphi \cdot dl = l2\pi.$$ (4)
Where $l$ is the topological charge, $\nabla \phi$ is the phase gradient and $\phi$ is the phase distribution of the vortex.

![Wavefronts of different optical vortex modes](image)

Figure 14 - wavefront, phase and intensity of different optical wavefronts

### 1.2.2. Optical vortex essentials

The propagation of an optical vortex in free space could be investigated by employing the optical vortex beam outer and inner radii at the source plane ($z = 0$) and utilizing the host Gaussian beam width ($w(z)$), which means that the vortex beams divergence could be analyzed.

Since phase singular beams possess OAM and because its modes have an unlimited dimensional orthogonal basis. To enhance the bandwidth in the communication field this OAM could be used as an information carrier.

For using fibers designed specifically for OAM modes and protocols with vortices, which means having both de-multiplexing and multiplexing setups for this modes, the intensity distribution of those OAM modes and their divergence must be known.

Under strong focusing circumstances the vortex beams spatial distribution has been investigated for which the maxima position changes linearly by the order instead of the order’s square root. By using of the radial alteration of intensity position, the vortex beam divergence in free space propagation has been theoretically investigated.

Lately, a theoretical studies revealed that, if the OV are produced by the use of pure mode converter, its divergence changes as the order square root. Yet, if they are created by the use of diffractive optical elements such as spatial light modula-
tors, their divergence will vary linearly with the order. Furthermore, the novel and commensurable parameters (the outer and the inner radii) could be used for investigating the vortex spatial profile. One of the properties of the outer and inner radii is that when the propagation distance is beyond Rayleigh range, they will alter linearly. Moreover, the outer and inner radii at \( z = 0 \) and the host Gaussian beam width at the observation plane controls the vortex at that plane.

If we assume that the vortex of order \( m \), attached in Gaussian host beam of width \( w_0 \) has a field distribution as:

\[
E_z(r) = (x + iy)^m e^{-\left(\frac{x^2+y^2}{w_0^2}\right)}.
\]  

(5)

And the intensity is:

\[
I_z(r) = r^2 e^{-\left(\frac{2r^2}{w_0^2}\right)}; \quad r^2 = x^2 + y^2.
\]  

(6)

The distribution of intensity and line profile is shown in figure 15.

![Figure 15](image)

Figure 15 - (a) The distribution of intensity and (b) line profile for an optical vortex beam of order 1

Here, two parameters of the optical vortex have been defined; or in other words, the inner \((r_1)\) and the outer radii \((r_2)\) have been defined, these two parameters corresponds to the radial distances at which the intensity is equal to 13.6% of the maximum intensity at \( r = r_0 \); The \( r_2 \) at the outer region of the vortex which the point distant from the origin, and \( r_1 \) is the point nearer to the center. If we assume that \( w_0 = 1 \), and the measuring radial distances unit is \( w_0 \), the inner and outer radii of optical vortex ring are given as follows:

\[
r_1(0) = \sqrt{\frac{l + 1.3 - (s_i)}{2}}.
\]  

(7)
\[ r_2(0) = \sqrt{\frac{(l + 1.3 + \sqrt{s_i})}{2}} \]  

\[ s_i = (l + 1.3)^2 - l^2 e^{\frac{-1A}{l}}. \]  

These radii coincide to the source plane \((z = 0)\) at which the vortices are being created. If the OV beam is propagating through free space, the variation of both the outer and inner radii related to the propagation can be investigated and studied, where the corresponding radii at \(z = 0\) controls the divergence, it has been noticed that the divergence is constant when \(z\) is large \((z >> z_R)\), and is given by:

\[ d_i = \frac{w_0 r_i(0)}{z_R}; \quad i = 1, 2. \]  

There are two categories of optical vortex beams: Vectorial vortex light and scalar optical vortex.

1.2.2.1 Vectorial vortex light

This type of optical vortex beams are distinguished from the other OV beams by the property of having polarization singularities, they are usually called Vectorial optical light beams \([17-20]\), which means that the state of polarization (SOP) of OV light and topological charge utilization should be taking into consideration in OAM characterization. The vector Bessel-Gauss mode is the most known class of modes of Vectorial vortex light which is distinguished by having cylindrical polarization symmetry property. Mathematically, the transverse magnetic field solution of the vector Bessel-Gauss mode with azimuthal polarization symmetry is described by the following formula:

\[
\bar{H}(r,z) = -H_0 J_1 \left( \frac{\alpha rz_0}{z_0 + iz} \right) \exp \left[ -\frac{i\alpha^2 z}{2k} \frac{2k}{z_0 + iz} \right] \bar{h}_\phi \exp[i(kz - wt)].
\]  

Where \(\alpha\) is a scale constant, \(J_1\) is the first order of Bessel function of the first kind, \(u(r, z)\) is the fundamental Gaussian solution, \(\bar{h}_\phi\) is the unit vector in the \(a\) \(H_0\) is a
constant magnetic field amplitude. Likewise, an electric transverse field solution should exist:

\[
\vec{E}(r, z) = E_0 J_1 \left( \frac{\alpha r z_0}{z_0 + iz} \right) \exp \left[ -\frac{i \alpha^2 z}{2k} \frac{2k}{z_0 + iz} \right] u(r, z) \exp \left[ i(kz - wt) \right] e_\phi.
\] (12)

A famous class of Vectorial vortex beams called cylindrical vector beams where its state of polarizations (SOPs) hold rotational symmetry. The majority of cylindrical vector beams (CVBs) own a radial or azimuthal polarization as shown in figure 16 ((a), (b)) [17-19]. Where at any point of the beam, the SOP follows a radial and azimuthal direction, respectively. The radial and azimuthal polarizations shape the basis for CVBs because of their orthogonality. The meaning of this is that by using these two basis (azimuthal and radial modes), any general cylindrical vector beams (CVBs) could always be represented by a mixture of azimuthal and radial modes as illustrated in figure 16.

Figure 16 - Cylindrical vector beams: (a) radial polarization; (b) azimuthal polarization; (c) generalized CVB
1.2.2.2. Scalar optical vortex

Unlike Vectorial vortex beams, the scalar optical vortex beams are beams with no polarization singularities. It can assumed that any optical vortex with \( e^{(-i\varphi)} \) wavefront that holds distinct orbital angular momentum of \( l \hbar \) per photon is a scalar optical vortex, where the topological charge is denoted \( l \) which means the OAM of vortex light beam is directly correlated to the topological charge \( l \) [21].

In free space, solving the scalar Helmholtz is required to achieve the solutions of the standard paraxial beams that has a harmonic temporal dependence:

\[
\nabla^2 E + k^2 E = 0. \tag{13}
\]

\( k = \frac{2\pi}{\lambda} \) is the wavenumber, in the Cartesian coordinates, and if it is assumed that the amplitude function \( u(x, y, z) \) z derivative has a slow-variation function of \( z \), which means:

\[
\left| \frac{k}{\partial z} \right| \gg \left| \frac{\partial^2 u}{\partial z^2} \right|. \tag{14}
\]

The Hermite-Gauss solution for HG modes is achieved if the separation of \( x, y \) variables was utilized:

\[
u(x,y,z) = E_0 \frac{w_0}{w(z)} H_l\left(\frac{\sqrt{2}x}{w(z)}\right) H_n\left(\frac{\sqrt{2}y}{w(z)}\right) e^{(i\frac{kr^2}{2q(z)})} e^{(-i\delta_l(z))}. \tag{15}\]

\( E_0 \) is the constant amplitude of the electric field, \( w_0 \) is the beam waist, \( w(z) \) is the beam size, \( z_0 \) is the Rayleigh range, \( H_m \) is the Hermit polynomials and finally \( q(z) \) is the complex parameter of the beam.

The Gouy phase shift is defined: \( \delta_{lm}(z) = (l + n + 1)\arctan(z/z_0) \).

When \( l = n = 0 \), the famous fundamental Gaussian beam solution is gotten.

In order to obtain the wanted Laguerre-gauss solution modes, it is important to solve the Helmholtz equation in the cylindrical coordinates, applying the approximation of the slow envelop variation and also using the \( r \) and \( \varphi \) variables separation. The obtained Laguerre-gauss solution modes are:

\[
u(r, \varphi, z) = E_0 \left(\frac{\sqrt{2}r}{w}\right)\frac{\sqrt{2r^2}}{w(z)} \frac{w_0}{w(z)} e^{(i\frac{kr^2}{2q(z)})} e^{(-i\delta_l(z))}. \tag{16}\]
Where $l$ is the topological charge, $p$ is the number of nodes in radial direction, $w(z)$ is the Gaussian beam size, $L_p^l(x)$ is the associated Laguerre polynomials.

$$z_0 = \frac{\pi w_0^2}{\lambda}.$$ Is the Rayleigh range;

$$\delta_p^l(z) = (2p + l + 1)\arctan\left(\frac{Z}{Z_0}\right).$$ Is the Gouy phase shift;

$$q(z) = z + iz_R.$$ is the complex beam parameter.

When $l = p = 0$, the result is the fundamental Gaussian beam.

When $l \neq 0$, the result is a Laguerre-Gauss mode that has a term of vortex phase equivalence to $e^{il\phi}$, where a linear phase increase in the azimuthal direction to the field is influenced by the phase profile of the transverse vortex represented by $\phi$.

The helical phase shape and the intensity distributions at the focal plane for LG$_{01}$, LG$_{11}$ and LG$_{02}$ modes of optical vortex is shown in the figure 17.

![Figure 17 - Intensity distributions (a, c, e) and corresponding helical phase pattern (b, d, f) at focal plane for LG$_{01}$, LG$_{11}$ and LG$_{02}$ mode of vortex beams](image)

**1.2.3. Orbital angular momentum of optical vortex**

A light can hold a spin angular momentum (polarization) or an orbital angular momentum, a light that has an OAM means that the light spatial distribution has an effect on the light angular momentum component if the light has an angular momentum component in the first place, if a light contain OAM, it can be categorized into two groups:
- The external orbital angular momentum: if the total linear momentum is cross product with center of the beam (position of the light beam) which means that the angular momentum is depending on the origin.
- The internal orbital angular momentum: the light wave is linked with the screwed or helical wavefront which means that the angular momentum is independent on the origin.

The phase spin velocity around the singularity is the physical meaning of the topological charge \( l \), where a counterclockwise rotation represents a positive topological charge (TC) and clockwise spin represents a negative TC [22].

The Light’s linear momentum azimuthal component need to be \( \hbar k_0 \lambda / 2\pi r \) if at the radius and with respect to the vortex axis, the pointing vector is \( \lambda / 2\pi r \). The beam’s angular momentum is equivalent to \( \hbar \) per photon if only the radius vector is multiplied by the azimuthal component. Where \( \hbar \) is Planck’s constant divided by \( 2\pi \) and the topological charge is denoted as \( l \).

1.2.3.1. Fractional topological charges of optical vortex

So far, we have been discussing optical vortices with integer topological charges and didn’t mansion the possibility of having an optical vortex with non-integer topological charges, there is a possibility of creating an optical vortex beam where the phase has an incomplete integer number of \( 2\pi \) revolutions, when this happens, a fractional optical vortex beam that holds a non-integer number of orbital angular momentum is obtained [23-24].

To keep the laws of nature intact, any the fractional optical vortex is the summation of integer topological charges and a sequence of alternating optical vortices or phase singularities that produce a dark line by cause of the edge dislocation of the vortex phase[25]. Because of that the fractional optical vortices propagation is variable and unstable, we can identify the fractional vortex propagation instability by observing both the phase profile and the intensity of it.

The superposition of light modes that have different values of topological charge \( l \) is one way for presenting mathematically the fractional optical vortices. If the
quantum notations are used; to describe a fractional vortex, the topological charge integer values are utilized as a basis set.

For instance, we can define the step of fractional phase as $|L(\alpha)|$, where $L = l + \gamma$ and $l$ is an integer and $\gamma$ is a non-integer number that is less than one and greater than zero. The equation below describes the fractional charge by using the topological charge integer values as basis set:

$$|L(\alpha)|=\sum_i c_i |L(\alpha)|/l).$$ (17)

Now, $c_i |L(\alpha)|$ is defined as:

$$c_i |L(\alpha)|=e^{i\gamma_0} \frac{ie^{i(L-l')\phi_0}}{2\pi(L-l')} [e^{i(L-l')\alpha}(1-e^{i2\gamma})].$$ (18)

To define the interval $\phi_0 \leq \theta < \phi_0 + 2\pi$, as a starting point the arbitrary angle $\phi_0$ is used. An important note that the fractional vortices are not totally characterized by $l$ when they are evaluated, so they are dependent on the edge dislocation orientation. This case doesn’t appear when dealing with optical vortices that contains an integer charge orbital angular momentum.

$\phi_0$ defines one basis set for the azimuthal angle even though it is an arbitrary angle, $\theta$. It is essential to be acknowledge that depending on the chosen value for $\phi_0$ the fractional vortex states are different.

![Figure 18 - Computer-generated binary grating with a half-cut of fringes along horizontal line.](image)

**1.2.4. Optical vortex generation methods**

Because of the significant role that optical vortices play in many applications, in the latest years, scientists and researchers have more concern about gener-
ating pure, efficient optical vortices using reliable methods, where using laser for
direct generation by implementing few modifications on the laser cavity to gain
optical vortices impeded in the laser beam. Yet, this technique has a many flaws as
it cannot be trusted and hard to be controlled, so myriad techniques and methods
have been developed to generate phase singularities such as using: Spiral phase
plate (SSPs), astigmatic mode converters, deformable mirrors, q-plates, S-plates,
Spatial light modulators, computer generated holograms, helical mirrors, dielectric
wedges [26-31]. Some of these techniques will be described below:

1.2.4.1. Deformable mirror

Deformable mirror (DM) (similarly know as adaptive mirror) is a dynamic
device contain a changeable thin mirror where its surface profile is modified using
the actuators connected in the rear side of this mirrors. Today, a lot of designs and
techniques used to create new types of DM, these types such as stacked actuator
deformable mirror [32], mirrors based on micro-electromechanical systems [33],
bimorph deformable mirrors [34] and the most famous and used are segmented
mirrors [29], are used mainly in astronomy and as in adaptive optics field as well
as they are lately used in ophthalmology [34]. Furthermore, the optical vortices can
be generated using deformable mirrors, however it is more advantageous to use
segmented DM rather than the usual continuous faceplate deformable mirror be-
cause the discontinuous line between the edge and the singularity is needed in the
DM surface which make the traditional continuous faceplate DM unsuitable for
generating the an optical vortex. Moreover, a vortex shape can be obtained with the
discontinuities between segments of DM and the beam reflected from the surface
gain a phase from this vortex shape segments. A full one wave will be obtained by
the reflected beam if a one-half of the wavelength of the input beam jump is made
by the surface at the discontinuity which will create phase singularity with topolog-
ical charge \( l=1 \), if the mechanical restriction of the DM taken into account, any
vortex charge can be generated only by multiplying the segments pistons amplitudes
and tilts. DM can be used at different wavelength due to the reflecting prop-
erty of the mirror surface. Another advantage of using segmented deformable mir-
ror (SDM) is that it can easily generate the fractional vortices. The figure 19 illustrate an Iris AO S37-X SDM.

Numerous advantages that a deformable mirrors can offer over other techniques such as: it is dynamic not static and it is a device that is independent on the polarization and wavelength of the input beam.

![Figure 19 - Iris AO S37-X SDM](image)

**1.2.4.2. Spatial light modulator**

When we talk about optical vortex generation, the most likely used method is by using computer generated hologram (CGH) or as known as the generation of optical vortices using spatial light modulator (SLM). The most common method for producing vortex beams in laboratory systems are the SLM and it is the method used in our experiments due to its simplicity of use and efficiency. SLM can be quickly programmed by using computer’s video interface because SLM is just a pixelated liquid crystal display device (LCD), elliptical crystals are aligned inside the LCD and by that they have the same direction orientation. When a voltage is applied, the ordinary and extraordinary axis will have different refractive indices because of the elliptical crystals are rotated due to applied voltage and because of that the angle of incidence controls the refraction index value that is seen by the incident beam.

So every single pixel can define a different refraction index because by controlling the voltage applied to every molecule, every molecule can be rotated independently.
Refractive and translucent states are the states that an SLM can either be, and that is respectively depends on whether the modulation of the incident beam waves by either reflecting off or passing through the SLM surface.

Twisted nematic and parallel aligned are the two main types of SLMs [29]. Unlike the twisted nematic SLMs, parallel aligned doesn’t have crossed polarizers and no control on the polarization whatsoever; the phase of the incident light is the only thing being controlled by the parallel aligned SLMs, nevertheless, when a voltage is applied across the LCD pixels, they rotate and the rotation causes a variation in the refraction index and by this way a modulation of the phase is allowed.

The polarization and the phase of the incident beam can be affected by twisted nematic LCDs. As it comes from its name the pixels in twisted nematic LCD twist in both transverse and optical axes. The phase modulation of the beam is controlled by the pixels having different refractive index, which in turn caused by the tilting and twisting.

The crossed polarizers located in either side of the LCD control the light polarization, the light is permitted to go through the second polarizer when no voltage is applied because the pixels are oriented in a manner that the phase is rotated 90 degrees. Now, if a voltage is applied, the light is partially blocked by the 2nd polarizer because the pixels rotate such a manner that the phase is shifted much less than 90 degrees. The pixels will continue to rotate until no light pass the second polarizer if more voltage is applied to the LCD.

![Figure 20 - Twisted nematic liquid crystal display](image)

The generation of optical vortex using SLM is based on the control and utilization of the computer and electronic crystal controlling the SLM, using a changing re-
fractive indices, as a spiral phase plates, or a fork let patterns, or even some similar patterns that have a topological charge that doesn’t equal to zero, a vortex beam can be created.

1.2.4.3. q-plate

A ±2q topological charge of an optical vortex can be produced based on the polarization of the input beam. It is well-known that the topological charge q at the photonic quantum vortex center defect if the local optical axis azimuthal distribution is taken into account. Where the q plates can be used inside the cavity of a laser for generating an efficient phase singularity. The basic operation of q plate is illustrated in the figure 21.

Figure 21 - Illustration of optical vortex generation using q plate
2. INTERFEROMETRY WITH OPTICAL VORTEX

2.1. Interferometry fundamentals

2.1.1. Definition

In the field of optics, in order to extract information from some systems, an electromagnetic waves are superimposed causing the interference phenomena, this principle is used by a group of methods and techniques called interferometry, interferometry is a technique used in countless fields such as engineering, astronomy, remote sensing and even in quantum physics where the system that detected gravitational waves was based on Michelson interferometer [36, 37].

2.1.2. Interferometry basics

The light behavior can be modelled either a stream of massless particles (photons) and a propagating electromagnetic waves, the former model is applied when dealing with quantum optics, while the latter is applied in the classical electromagnetism.

a) Interference

When two or more optical beams intersect, the intensity pattern resulted is unlike any of the individual intensity patterns. This phenomena is known as optical interference. The resulting intensity profile is just the summation of all the profiles of intensity at every point that is because all the interfering waves’ intensities are added together. If the waves continue propagating after the plane where the interference happened, the interference will not affect any of this waves that is because of the property of wave-like nature that the light holds. An incoherent light is a light that its phase difference is changing with time. However, if the interference pattern will emerge rather than interfere, the interfering light is said to be coherent. A coherent light is where all electromagnetic waves phases are identical at every point on a line normal to the propagation direction, where a laser is known to produce a monochromatic coherent light.

If considering an interference of two monochromatic waves with equal wavelength and have the same polarization, where their complex amplitudes are:
\[ U_1(x, y, z) = u_1 e^{i\phi_1}. \]  
\[ U_2(x, y, z) = u_2 e^{i\phi_2}. \]

The sum of the individual amplitudes represents the resulting complex amplitude where its intensity can be written as:

\[
I = |U_1 + U_2|^2 = u_1^2 + u_2^2 + 2u_1u_2\cos(\theta_1 - \theta_2);
\]

\[
= I_1 + I_2 + 2\sqrt{I_1I_2}\cos(\Delta \theta).
\]

Where \(I_1, I_2\) are the individual intensities, the maximum intensity is reached when the phase difference between the two waves \(\Delta \theta\) is a multiple of \(2\pi\):

\[
\Delta \theta = m2\pi. \quad \text{for} \quad m = 0, 1, 2, \ldots
\]

This is known as constructive interference and the destructive interference when the minimum intensity is reached when:

\[
\Delta \theta = (m + 1)2\pi. \quad \text{for} \quad m = 0, 1, 2, \ldots
\]

Where \(m\) is an integer known as the interference order. The interference pattern contains a sequence of dark and bright lines called “fringes” as a result of this destructive and constructive interferences.

**b) Diffraction**

To study the behavior of light and calculate the interference pattern at several points, the diffraction theory is applied. Depending on the length the light beam has traveled, the diffraction patterns past the aperture are expressed with three different regions: Rayleigh-Sommerfeld, Fresnel and Fraunhofer [35]. Figure 22 demonstrates these three regions. The Rayleigh-Sommerfeld region starts after the aperture, while the Fresnel region start after the light propagate various wavelengths. At last, the Fraunhofer region will start when the light travels even further beyond Fresnel region limit. The Huygens Principle can explain quantitatively the diffraction theory as: each point of a wavefront could be counted as a source point for secondary spherical waves. The wave field at any other point is the coherent superposition of these secondary waves, where the diffraction is expressed quantitatively by the Fresnel-Kirchhoff integral:
\( \Gamma(\xi',\eta') = \frac{i}{\lambda} \int \int_{-\infty}^{\infty} \frac{\exp(-i\frac{2\pi}{\lambda} \rho)}{\rho} U(x,y) B \, dx \, dy. \) \quad (24)

Where

\[ \rho = \sqrt{(x - \xi') + (y - \eta') + d^2}. \] \quad (25)

\[ B = \frac{(\cos \theta + \cos \theta')}{2}. \] \quad (26)

U(x,y) is the complex amplitude in the plane of the diffracting aperture. \( \Gamma(\xi',\eta') \) is the complex amplitude in the observation plane, \( (\xi',\eta') \) is the position of the diffraction plane where the field is proportional to the field at the entrance side of the aperture, \( \rho \) is the distance between a point in the observation plane and a point in the aperture plane. B factor is introduced to the Fresnel-Kirchhoff integral to eliminate the unrealistic situation where the Huygens principle illustrates that the secondary wave propagate back to the source and not only in the forward path, whereas the experiment shows that the wavefronts are always propagating in one way. Where \( \theta \) is the angle between the unit \( \vec{m} \) vector perpendicular to the aperture plane and the incident light from the source. \( \theta' \) is the angle between \( \vec{m} \) and the diffracted light. The figure 23 illustrates the Huygens principle.

![Figure 22 - Illustration of Rayleigh-Sommerfeld, Fresnel, Fraunhofer regions](image)
2.2. Some of Interferometry types

Two or more light beams, obtained from the same source however moving along separate pathways which are made to be interfered are required in any optical arrangement for interferometric measurement. Interferometers could be categorized as two-beam interferometers such as Michelson interferometer, Sagnac interferometer, and shearing interferometer or multiple-beam interferometers such as Fabry-Perot, three beam and double-passed two-beam interferometers [36, 37]. In this section some types of interferometers are going to be discussed.

2.2.1. Michelson interferometer

The most known type of an optical interferometry configurations is the Michelson interferometer (MI), where it has been used as essential part in the system that detected gravitational waves [37], the Michelson interferometer uses beam splitter to split a monochromatic light then join the two beams to generate interference fringes. One of the beams traverses the beam splitter only once while the other traverses it three times. Now to equalize the optical paths in glass, a compensating plate with an equal thickness of the beam splitter is introduced to the second light. The first beam hits a fixed mirror and the other beam strikes a transportable mirror. An equal thickness fringes are attained if the input light was collimated. If the portable mirror moves, the fringes also moves in the same direction as the mirror did.
By moving the mirror of the Michelson interferometer and counting the interference fringes that moves by a reference point, an accurate distance measurement can be obtained, where the distance “d” that the movable mirror traverses is linked to the “m” fringes by the following:

\[ d = m \frac{\lambda}{2}. \]  

The Michelson interferometer arrangement is shown in the figure 24.

![Michelson interferometer diagram](image)

Figure 24 - The basic arrangement of the Michelson interferometer

### 2.2.2. Holographic interferometry

Holographic interferometry (HI) is a technique used for measuring the length changes of the optical path caused by transparent media refractive index variations or opaque bodies’ deformations such as gases or fluids. By using HI, optical path variations of one hundredth or less of the used beam wavelength can be measured. What makes HI so spatial is that it is a nondestructive, non-contact metrological method and has high-level of measurement sensitivity. There are many types of HI such as phase shifting, double-exposure, real-time, time-average HI and refractive index measurement using HI.

- Phase shifting HI (PSHI): is a technique utilized to determine the phase of the interference by recording extra information [36], PSHI principle is that multiple interference patterns that have mutual phase shifts are recorded and phase shift is presented in the reconstructed hologram.
- Refractive index Measurement by HI: it is a method used for detecting the refractive index variations within transparent media, this technique is mainly used to determine the fluid or gaseous media’s concentration or temperature variations where a phase variation occur between two waves going through the medium before and after the change of optical path length that is caused by transparent medium refractive index variation.

- Double exposure holographic interferometry (DEHI) [36]: DEHI use the same recording plate to compare two wavefronts, this is done by recording holographically the image of the test object in same holoplate in two separated times, if the object is spatially moved between the separated time intervals, the final image in the holoplate will appear to have interference fringe system which is linked to deformations that happened to test object surface caused by the disruption. However, if nothing alters the object wavefront during the two exposures, a single-exposure hologram will appear on the holoplate.

- Real-time holographic interferometry (RTHI) [36]: in RTHI and after processing the first exposure, the holoplate is placed in the original recording location. If the hologram is illuminated with reference light, the object’s light is overlapped with the virtual reconstructed 3D image and superimposed upon it. The actual object and the holographically reconstructed reference object waves causes phase changes which lead to interference patterns in the holoplate, these interference patterns can be observed in real time.

### 2.2.3. Shearing interferometry

The wavefront under test’s amplitude is divided to two beams when the shearing interferometers are used. One half is unaffected, while a significant parameter is changed in the second half. Shearing interferometers (SIs) do not need a reference wavefront, which is the most important feature of SIs, because the wavefront under examination is compared with its replica. The interference pattern is formed by those parts of the wavefronts that holds the same polarization which are
joined in the plane of detection. It has been shown that the obtained interference pattern with SI under small shear approximation represents the phase distribution first derivative (gradient) instead of the phase function itself. Three basic types of shearing interferometers that exist are: the radial, the lateral and the rotational. The wavefront inversion interferometer or a reversal shearing interferometer has been proven to be a special case of the rotational shearing interferometer; whenever the shearing angle $\phi = 180^\circ$.

In the field of optics, shearing interferometers are widely utilized for myriad number of applications such as [38]: testing of beam collimation, optical component testing, ophthalmic lenses power distribution measurement. Furthermore, they are used in the detection of wavefront aberration in optical beams and its real time measurements in the adaptive optics field. Figure 25 illustrate the basic wavefront operations of rotational and reversal shearing interferometers.

![Figure 25 - The basic wavefront operations of rotational and reversal shearing interferometers](image)

### 2.2.3.1. Lateral shearing interferometer

In the first half of the 20th century, the Italian scientist Ronchi was the first one to present laterally sheared wavefronts for testing optical components. The lateral shearing interferometers are the most broadly used SI configuration in the optics field. In LSI, the test wavefront amplitude is divided and shifted spatially in the transverse plane to create interference fringes in the overlap region of the test wavefront and its replica [39]. The shear plate is the most frequent configuration used by LSI, which is a thick, plane parallel plate. Moreover, when the shear plate
is stricken by the test wavefront at an oblique angle, the light beams reflected from shear plate’s front and back surfaces are laterally displaced and in the overlap region the interference fringes are created.

Now, if the wavefront is almost planar, by displacing the wavefront in its own plane, the lateral shear is attained. However, when the wavefront is practically spherical, by sliding the wavefront along itself by rotation around an axis that goes via the curvature center of the spherical wavefront, the lateral shear again is attained. The figure 26 demonstrates schematically the shearing interferometry principle.

The difference between the desired wavefront and the actual one is the wavefront error $W(x,y)$. If it is assumed that the wavefront used is almost planar, the error at every spot on the sheared wavefront is $W(x-S,y)$ if only when the wavefront is sheared by a quantity $S$ in the $x$ direction. When the displacement $S$ equals to zero, no wavefront difference is found anywhere in the interferometer. Now, at a random point, $\Delta W(x,y)$ which is the resulting wavefront difference between the sheared and original wavefront can be expressed in terms of the wavefront wavelength:

$$\Delta W = W(x, y) - W(x - S, y) = n\lambda.$$  \hspace{1cm} (28)

Where $\lambda$ is the wavefront wavelength and $n$ is the order of the interference fringe. The figure 27 shows a shear plate interference.
2.2.3.3. Rotational shearing interferometer

The interferometer that run the operation of revolving one wavefront with respect to the other which is its replica, is called a rotational shearing interferometer. In the rotational shearing interferometer, one pupil image is rotated by a small angle about the optical axis with respect to its replica (reference beam). The two images will overlap if the center of the pupil is coincided with the rotation axis. A different spatial frequency is measured by each baseline, mapping the entire plane of the frequency up to a cut-off frequency [40].

In the rotational shearing interferometers, an interference between two identical wavefronts happens. However, it is attained by a particular angle about these two wavefronts, one of them is revolved with respect to the other. Let us introduce the wavefront by \( W(\rho, \theta) \), the interferogram of RtSI is presented as:

\[
\text{OPD}(\rho, \theta) = W(\rho, \theta - \frac{\phi}{2}) - W(\rho, \theta + \frac{\phi}{2}).
\]  

The rotation of one wavefront with respect to the other beam is denoted by \( \phi \). OPD is the optical path difference, which is the function of refractive indices that are traversed. One property of rotational shearing interferometer is that the RtSI is not sensitive to real defocusing. Furthermore, the RtSI angle \( \phi \), controls whether a RtSI can detect astigmatism and coma or not. There are many ways to obtain a rotational shearing interferometer by using a conventional optical components and
interferometers, $180^\circ$ RtSI can be obtained using a Twyman-Green interferometer or base on cyclic or Sagnac interferometers, where a $\phi/4$ generate a RtSI angle $\phi$, inside the interferometer closed path, Dove prisms also can be used to obtain a RtSI, just like in the case of Murty and Hagerott’s RtSI.

The most effective and easiest to implement is the RtSI developed by Roddier which is called Variable rotational shearing interferometer which is based on Michelson interferometer, where a right angle prisms replace the reflecting mirrors and in MI [40]. Also, a cubic beam splitter replace both the compensating plate and half silvered plate BS. In the resulting RtSI, the RtSI with shear angle $\phi = 0^\circ$ resembles the Michelson’s interferometer. By rotating the right angle prism, the wavefront from one arm of the RtSI is rotated from $0^\circ$ to $90^\circ$. Furthermore, the wavefront is also flipped from left to right by the prism. Figures (28, 29) represents RtSI developed by Murty and Hagerott and Roddier’s VRtSI.

Figure 28 - Murty and Hagerott’s Rotational shearing interferometer

Figure 29 - Variable rotational shearing interferometer
2.2.3.2. Reversal shearing interferometer

In the reversal shearing interferometer (RvSI) two wavefronts are produced where the any deformations or distortions on one wavefront are symmetrical with respect to those on the other one where the symmetry axis is the diameter. In other words, the wavefront reversal around an arbitrary axis is equal to reversion around the x-axis pursued be a lateral shear s in the y-direction. The wavefront reversal around a reversing axis is demonstrated in figure 30 where point P goes to P’, the transforming equations shows that:

\[ \rho' \sin \theta' = \rho \sin \theta. \]  
\[ \rho' \sin \theta' = S - \rho \sin \theta. \]

In this case, the RvSI interferogram is:

\[ \text{OPD} = W(\rho, \theta) - W(\rho', \theta'). \]

It was proven that the RvSI is insensitive to symmetrical aberrations such as spherical aberration, astigmatism and defocusing. As illustrated in figure 30, in S=0 where the x axis coincides with the reversion axis. The symmetrical aberrations sensitivity and the interferometric pattern of this RvSI are the same to those of a LSI with shear S when the axis of reversion is moved at distance S/2.

Throughout the years, many techniques and methods were developed to create new configurations for RvSI, many of those interferometers where based on Köster prisms, like the one shown in the figure 31.
Another configuration was suggested by gates [41], which used the same principle as the previous one, however this RvSI can test only optical systems that has small numerical apertures. Gates RvSI is illustrated in figure 32.

In our project, the RvSI used is based on the variable RtSI, where the rotated prism is simply replaced with a reflecting mirror and the new configuration becomes an RvSI.

2.3. OAM of light measurement

In many applications concerning the optical vortices, it is crucial to know the used OV’s topological charge which denotes as the light propagate, the number of times a $2\pi$ completion around a closed loop the phase done. Many techniques and a straight forward methods have been developed for the determination of topological charges of OV light such as: triangular apertures, use of double slit interference, and shearing interferometers. The topological charge is effortlessly spotted by the naked eye, only by observing the interference fringes created by these techniques and methods.
The first method for OV’s topological charge measuring is by implementing the triangular aperture technique, when a light contains a vortex hits a triangular aperture, the diffraction pattern of the far field consists of truncated optical lattice [42]. The topological charge $l$ is directly linked to the amount of spots enclosed in generated diffraction pattern. Which means a higher or a lower topological charge is correlated to greater or fewer amount of spots, respectively. The topological charge is defined by the amount of spots produced along one of the triangle’s edges. Such as $l = N-1$, where the amount of spots calculated along one of the triangle’s edges is $N$, that means if four spots are created along edge of the triangle, the input light has a topological charge $l = 3$.

If the amount of spots presented in each triangle’s edge is linked to the TC, also the spot’s total number $M_i$, where:

$$M_i = \sum_{k=0}^{i+1} k = \frac{(i+1)(i+2)}{2}.$$  \hspace{1cm} (33)

$$l = \frac{(1 + 8M_i)^{1/2} - 3}{2}.$$  \hspace{1cm} (34)

The equation (34) shows that, for each spot’s total number there is an exclusive topological charge.

Another method for measuring the OAM of light is when the emerging beam’s (OV) phase pattern is interfered with an expanded reference beam which is the same beam used to generate the OV. In the fringe pattern, OVs look as forks in the interference pattern; Furthermore, the topological charge $l$ can be read easily by the dislocation of the fork in the fringe shape. Whereas, the amount of fringes gained or lost in the round path is counted when going on a closed orbit around the phase singularity. Where in the fringes advancing in one way (right to left) calculated oppositely with those coming in the reverse way. The singularity’s charge is given by the amount of fringes lost or gained. Whereas, the simplest law to follow is that for a single fork charge is the number of tines subtracted by one. Figure 33 shows the topological charge determination method in a fringe pattern.
The third method of the measurement of optical vortex topological charge is by using shearing interferometer, in our case by using rotational shearing interferometer and reversal shearing interferometer, as discussed in part 2.1.2. The phase gradients in the test wavefront are represented directly in the resulting interference fringes of the shear interferometer. Because of this property, the determination of if the beam contains aberration or it is collimated is obtained by using the shearing interferometer. Moreover, a straight fringes parallel to the shear’s direction is obtained in the fringe pattern if the light is collimated [43]. The effectiveness of shearing interferometers in the measurement of OV’s topological charge have been confirmed due to their sensitiveness to light’ phase gradient.

2.4. Shearograms

The shearograms are the phase distribution’s space derivatives, however, the shearograms do not represent phase gradients if the vortices are presented. As it is discussed above in the SI and in the transverse plane, the test wavefront is spatially moved and divided in amplitude to create fringes of interference by the wavefront and its displaced replica region of overlap, the localized phase gradients or the test wavefront’s wavefront slopes are represented in the fringes are in such an arrangement. The shear which can be lateral, rotational or radial. Now, for the fringes to be able to represent the phase gradient, the shear magnitude has to be small, based on the equation (28), $\Delta \phi$ is related to $\Delta W$ by:

$$\nabla \phi = \lim_{\Delta x \to 0} (k \frac{\Delta W}{\Delta x}).$$

(35)
If the shear $\Delta x$ is small, we can say:

$$\nabla \phi_x = k \frac{\Delta W}{\Delta x}. \quad (36)$$

Where $k$ is the propagation constant. If $|\nabla \phi|$ is small the above relation is accepted even if the shear $\Delta x$ is larger. When a test wavefront which is incident on the parallel plate at tilted angle gets split in amplitude at the two faces of the plate and the two reflected beams are laterally sheared interference pattern is as shown in the figure 34.

The plane parallel plate is utilized in such a manner that there is near normal incidence and the shearing beam is propagating back almost in the incoming beam direction.

Figure 34 - Shearograms when shear is small and defocus is not null (a) x shear and (b) y shear

When the phase singularities are presented in the SI the resulted shearograms do not represent true phase gradient [38].
3. OPTICAL VORTICES IN INTERFEROMETRY: SETUP AND RESULTS

3.1. Experimental setup

The aim of these experiments was to observe the interference pattern when the optical vortex interferes with itself by using shearing interferometers and determine the topological charge of the beam for integer charges by utilizing these interferometer arrangements. Our work consists of three sub-experiments, each one of these experiments has its own design, aim as well as results. First we start by discussing the method that has been used to generate the “fork shape” of optical vortex. After that we go directly to the experiment of using rotational shearing interferometry in optical vortices. Finally, we finish with the experiment of utilizing reversal shearing interferometry in optical vortices.

3.1.1. Optical vortex generation using CGH

One of the methods for generating optical vortices that haven’t been mentioned in the first chapter is the computer generated hologram method. This technique is the most used technique for OV generation, because of its simplicity and flexibility which means it can generate any type of optical field in an impressive speed [30]. Among the advantages that CGH offer over other methods is that it rarely create technical problems that delay the OV generation operation such as the coherence of the source or surrounding circumstances.

For generating an optical vortex light by using CGH, the technique is based on the principle of holography, where it is enough to compute the pattern of interference numerically using a mathematical software and print it physically. Moreover, the reference beam control the resulting holographic pattern when the interference is computed between the vortex and the reference. A fork shaping pattern that has a dislocation at the center will be present in the interference pattern if the reference wave is plane wave, and it is called the fork hologram (FH). However, a spiral shape [30] just as presented in the figure 35.b will be demonstrated at the interference pattern if a spherical wave is used as reference light.
Because of the grating structure of the fork hologram, unlimited amount of diffracted orders will be created if the fork hologram is crossed by a monochromatic light that has a wavelength $\lambda$, where the angle $\phi_l$ which is the angle between the $l$th order direction and the input light beam propagation axis is given by:

$$\phi_l = \sin^{-1}(\frac{\lambda}{\Lambda}).$$  \hspace{1cm} (37)

Where $\Lambda$ is the spatial fissure between the grating grooves far apart from the midpoint.

From now on, only fork holograms are in our interest, spiral holograms are less useful for TC determination than the fork holograms. Moreover, for generating an optical vortex with charge $l$, the associated field of this optical vortex $A_o$ is given:

$$A_o = I_o \exp(i\delta).$$ \hspace{1cm} (38)

Where

$$\delta = \tan^{-1}(\frac{y}{x}).$$ \hspace{1cm} (39)

$I_o$ is the real factor of the amplitude, $\delta$ is the azimuthal angle in the ($x$-$y$) which is orthogonal to the $z$-axis, now assuming that a tilted plane wave has vector $u$ on the ($x$-$z$) axis that creates an angle $\gamma$ with $z$-axis and has an invariable amplitude. If we keep assuming that the $I_r = I_o = I$, the computed interference between them which is the printed interference pattern from which we attain the fork hologram is:

$$T = |A_o + A_r|^2 = 2I \left[1 + \cos\left(\frac{L_\| + 2\pi x}{\Lambda}\right)\right].$$ \hspace{1cm} (40)
Multiple fork shape interference patterns are illustrated in the figure 36.

(a)                                                                   (b)

(c)

Figure 36 - Multiple fork shape interference patterns where at (a) \( l = -4 \), (b) \( l = -1 \), (c) \( l = 7 \)

Now, by calculating fork shape interference pattern equation, any pattern for any topological charge can be generated by using just a computer software such as Matlab (Appendix A) which make simple and effective. So in our work, we are going to depend on not on the analog method but the digital method for generating the vortices, because it is the most accurate and safest method for the generation.

Figure 37 - Scheme of optical vortex generation experiment setup
The arrangement in the figure 37 which is the same arrangement that have been made in the laboratory is the typical system for generating a good quality optical vortex, it consists of: Laser (1), a collimator (2), Spatial light modulator (SLM) (3), personal computer (4), lens (5, 7), Diaphragm (6) and a detector (8) which is in this case is a Camera Canon 5D Mark III, \( L_1 \) and \( L_2 \) are the lenses focal lengths, where \( L_1 = L_2 = 50 \text{mm} \), the laser is a 5mW He-Ne laser with a \( \lambda = 6.328 \text{ nm} \) wavelength, the two dimensional SLM used is a Holoeye transmissive LC-2002 spatial light modulator, the actual system is demonstrated in the figure 38.

Figure 38 - The real system in the optical vortex generation experiment

After the computation of the interference pattern utilizing a mathematical software such as Matlab (Appendix A), a picture of the fork shaped hologram is attained, by using the convenient kits to link the SLM with the PC, we transmit the picture electrically to the SLM that behaves as a transparency controller. The collimated light illuminate the SLM (the fork hologram interference pattern). Now, the transferred diffracted light pass through a lens that has a focal length of 500 mm, which is the same distance at where precisely the diaphragm is placed to keep only the first order and remove all the other orders, the beam that has the first order continue its track to the second lens and to the detector.

If the topological charge \( l = 2 \), the figure 39 shows the image obtained from the camera.
If the diaphragm is not used, all the orders will be presented after the second lens.

3.1.2. Interferometry setup

There are two interferometers are used to detect TC in the following experiments, the RtSI and RvSI, where in the first experiment just as presented in the figure 40, a rotational shearing interferometer is used, and as the same procedure as in section 3.1.1 and up to where the light that holds the vortex comes from the second lens. In this experiment, the vortex light doesn’t go directly to the detector. Instead, the light beam coming out from the second lens is split into two beams by a cubic beam splitter CBS (8), the first one goes to a fixed prism (10) and it is called the reference beam and return back to the CBS, the second one goes to a rotated prism (9) where it is rotated and sent back to the CBS, where it interferes with reference beam, the resulting interferometric pattern is registered by utilizing the detector (Camera), this experiment is repeated many times depending on the rotating angle of the rotated prism. In our case, to see the effect of the prism’s rotation angle on the incident vortex light, measurements were taken for the rotation angle $\varphi = 0$, then $10^\circ$, $20^\circ$, $30^\circ$ in the CW, then $20^\circ$, $30^\circ$ CCW and with many vortices that have different integer charges, the results are shown in the section 3.2.1.

![Figure 40 - Scheme of the optical vortices in rotational shearing interferometry experimental setup](image-url)
The actual setup of the OVs in RtSI experiment is illustrated in the figure 41.

In the second experiment, where instead of using RtSI, a reversal shearing interferometer is used to detect the vortex topological charge.

The same arrangement is used here as in the last part. However, the rotational prism is replaced by a reflecting mirror (9), where the light coming out from the second mirror continue its path till the CBS and splits into two waves, the first wave goes to the mirror and reflected back to CBS, the second beam travels to the prism, get reversed in accordance to the vertical axis of the prism and get back to CBS where it interferes with the first one and the detector registers the interferometric patterns. Both figures 42 and 43 shows the schematic and actual arrangements for optical vortices in reversal shearing interferometry experiment.

Figure 41 - The actual system used in the optical vortex in RtSI experiment

Figure 42 - Scheme of the optical vortices in reversal shearing interferometry experimental setup
3.2. Experimental results and discussion

While studying the OVs in interferometry, the fork shape shearograms have been the best to be used in the detection of topological charges of OVs.

3.2.1. Rotational Shearing interferometer fringes

The results of using RtSI for the detection of OV topological charges or in other words the amount of OAM that the OV holds, are presented in this section. As for the rotation angle $\phi$, when it equals to zero. The obtained fringes are illustrated in the figure 44.

Now, for the angles $\phi = 10^\circ, 20^\circ, 30^\circ$ in the CW, the acquired interference pattern fringes are demonstrated in the figures 45, 46 and 47, respectively.
Figure 45 - Experimental interference fringes generated by using OVs in RtSI when $\varphi = 10^\circ$ in the CW: (a) $l=2$, (b) $l=-2$, (c) $l=4$, (d) $l=-4$

Figure 46 - Experimental interference fringes generated by using OVs in RtSI when $\varphi = 20^\circ$ in the CW: (a) $l=2$, (b) $l=-2$, (c) $l=4$, (d) $l=-4$
Figure 47 - Experimental interference fringes generated by using OVs in RtSI when $\phi = 30^\circ$ in the CW: (a) $l=2$, (b) $l=-2$, (c) $l=4$, (d) $l=-4$

First of all, the fringes are a perfect double forked pattern that have a number of prongs $P$, which corresponds to the vortex charge, $l$ plus one, where $l$ is the absolute value of TC, so:

$$P = l + 1.$$  \hspace{1cm} (41)

Or by the conversion, where the vortex charge is equal to the amount of prongs in the interference pattern subtracted by one.

$$l = P - 1.$$  \hspace{1cm} (42)

Also, the second method for determining the OAM amount mentioned in section 2.3 and as shown in figure 33, can be applied easily for any fork shape in the attained results. Another remarkable difference between the pattern of fringes of the negative charges and the positive ones is the forks direction is reversed, however, while the forks orientation is reversed, the amount of prongs that every fork holds is still equivalent to the charge added by one. Determining the charge sign becomes very easy because the interference pattern of negative vortex charge is the mirror image of the interference pattern of the positive charge. Moreover, until now, the only charges clearly presented in the RtSI are the even ones.
Now, for the angles $\varphi = 20^\circ, 30^\circ$ in the CCW, the related interference pattern fringes are demonstrated in the figures 48 and 49 respectively.

Figure 48 - Experimental interference fringes generated by using OVs in RtSI when $\varphi = 20^\circ$ in the CCW: (a) $l=2$, (b) $l=-2$, (c) $l=4$, (d) $l=-4$

Figure 49 - Experimental interference fringes generated by using OVs in RtSI when $\varphi = 30^\circ$ in the CCW: (a) $l=2$, (b) $l=-2$, (c) $l=4$, (d) $l=-4
The results obtained look very similar to their CW peers. Yet, they are oriented and the TCs can be obtained easily.

The figure 50 reveal the achieved interference pattern fringes when the incident vortex light has odd integer number of charges when the prism rotation angle $\phi = 20^\circ$ in the CW.

![Figure 50](image)

Figure 50 - Experimental interference fringes generated by using OVs in RtSI when $\phi = 20^\circ$ in the CW: (a) $l=3$, (b) $l=-3$, (c) $l=5$, (d) $l=-5$

The vortex presence in the system is confirmed by the fork shaped fringes in the achieved interference pattern fringes. Nevertheless, because of the deformations on the prongs of the Forks, the determination of the vortex charge $l$ by using the equation (42) is hard in this case and it may be concluded that in RtSI case, the use of Vortices with even TC is better.

3.2.2. Reversal shearing interferometer fringes

In the second optical interferometry arrangements, the interference fringes that are generated by interfering two vortex beams were observed. As described in the experimental setup in section 3.2.1, the collimated beam was modulated in the SLM and then goes to the prism where it split to two waves, the first one goes to the mirror and return back, while the other goes to a prism where it is reversed and
return back to the cubic beam splitter where they interfere, from there the resulting light goes directly toward the detector which in our case is a Canon 5D mark III where the interference fringes could be viewed. Figure 51 displays the interference pattern fringes of OVs with charges 1 through 5.

Figure 51 - Experimental interference fringes generated by using OVs in RvSI: (a) \( l=1 \), (b) \( l=2 \), (c) \( l=3 \), (d) \( l=4 \), (e) \( l=5 \)

Now, at the first observation of the interferometric pattern fringes. If the observer uses the equation (42) to obtain the TCs of the input OVs, the attained results are the double of the actual ones that is because when the vortex beam is propagating in the prism, it gets reversed according to the prism’s vertical axis. So in the case of RvSI, a modified version of equation (42) is applied:

\[
 l = \frac{P - 1}{2} \tag{43}
\]

If the input OVs have a negative TCs, the interference pattern fringes reached are presented in the figure 52.
The point worth mentioning here is that the interference pattern fringes or the fork shapes of the negative vortex charges are just the horizontally flipped image of their positive peers.

As a conclusion, the experiments done proves that the detection of the amount of orbital angular momentum in OVs can be done by using either rotational or reversal shearing interferometers. In the end, the purpose of this work has been achieved.
4. SAFETY

The Laser which is an instrument that produces light radiation based of stimulated emission of electromagnetic waves, where the word “Laser” is an abbreviation of “Light Amplification by the Stimulated Emission of Radiation. The Light produced form Laser is under the type of optical non-ionizing emission, which differ from other types of light radiations (lamp or sun radiation) by having a distinctive properties such as monochromaticity (a fine bandwidth), all its waves are in phase (spatial coherence). Also, a high collimation which means that when the distance is increasing, the light does not expand remarkably as well (i.e small angular divergence), and by the combination of this features the laser radiation is discriminated from other light radiation sources.

The invention of Laser was by the hands of Theodore H. Maiman in 1960, since that myriad forms and shapes have been created and applied in almost every area in science and life in our time, from the usual shopping checkouts (code bar scanners) to medicine, education and industry, even it is used in research in the quantum physics fields (gravitational waves detection using laser).

One of the features of laser is that it releases a thin concentrated and powerful radiation rays, it can visible or not visible (i.e ultraviolet and infrared radiations) to human eye depending on the Laser wavelength. Because of this feature, the usage of lasers may create an optical and skin hazards and threats, however this dangers are dependable on the wavelength and power of the output.

The threats of lasers are frequently concluded to the capability to burn skin and to damage the eyesight of the human being. However, usually the ones that cause the greatest risks are not the radiation or optical hazards but most risks come from chemical dyes, cryogenic liquids and electrical supplies.

A legal requirements has to be followed by the users which let them define Hazards and take suitable actions to consider, eliminate or at least reduce those risks (nonoptical and optical). It is mandatory for every user at research lab or anywhere else to protect themselves and others and take health and safety precautions from potential hazards to guarantee a safe environment.
Now, to reduce hazards and ensure a secure work atmosphere, an assessment methodology was set out. Whereas, a number of different regulation such as PPe regulations, Electricity at work and CoSHH (control of substances hazardous to health) are set to assess the non-optical hazards.

In the United Kingdom, the workers protection from health threats and Dangerous sources of artificial optical radiation (AOR) including laser radiation is required, it was described in the regulations of 2010 (AOR). The obligations to control the threats from light radiated from all artificial sources in all its types (visible or not visible) where covered in those regulations.

Moreover, the safety of laser products was reported in BS EN 60825 Series of documents of the British Standards Institution (BSI). The standards for manufactures on Fiber optics systems, free-space communication systems and lasers are covered in the corresponding International Electrotechnical Commission’s IEC documents.

4.1. Environmental conditions and some Hazardous factors description

Since laboratories dealing with Lasers contains non-optical equipements, plenty of factors (apart from Laser emission) can create risks when dealing optical systems that are based on laser radiation, these risks must be identified and must be controlled by the help of the manufacturer’s safety guidance. The non-optical threats that may danger the laboratory personnel which in fact are not taken into considerations and dealt seriously include: fume, dangerous substances, fire, mechanical, electrical, collateral emission and other important environmental conditions such as humidity and temperature.

4.1.1. Atmospheric influences

When dealing with high-power beams, the most hazardous factors are the atmospheric conditions, such examples for this conditions is that a higher power laser rays can ignite combustible materials and gases, solvent vapor or even dust that are present in the environment which may lead to explosions.

4.1.2. Vibration and mechanical stun

The operation of the laser system is sensitive by external disturbances such as a mechanical stun or vibrations which may influence the operation of the laser
and this can create a misalignment of the optical pathway, which in many cases leads to a dangerous stray beams.

4.1.3. Temperature and humidity

Among the sensitive causes that keep the user in good health and secure during the usage of lasers, the temperature and the humidity conditions are the most factors that cause a disturbance in the functioning of the laser, where any extreme high or low ambient temperatures, or high levels of humidity can affect the performance of the laser equipment which leads to unpredictable dangers.

4.1.4. Power supply disturbance or interruption

The disturbance (instability) or disconnection of the electrical supply is one the factors that is not taken seriously when dealing with laser systems, this interruptions might lead to either hazardous on lab personnel or systematic errors.

4.1.5. Electromagnetic interference and computer problems

The interference between electromagnetic (magnetic or electric radiations) or even high pulses created by the data or supply cables with the laser gadgets can create an unwanted effects. Moreover, when a laser protective systems are partially or completely operating by a software control, a glitch in such software can create a catastrophic results.

4.1.6. Human-factor

In any system that a human is involved, there is always misalignment and other factors that can create threats, where the misalignment of the physical arrangement of the laser equipment plus the lack of space will increase the possibility of accidents which effect that individual’s safety.

Now, it is safer to take into consideration separately the probability of exposure of users to dangerous laser beams because it is affected by the laser in the experiment. Based on the international standard of experimental condition, when you are doing your research it is obligatory to concentrate on the environmental aspect to get the wanted results without any disruption on the objective prospect. Many conditions must be taken into account when dealing with a special experiment of laser measurement. Moreover, when dealing with high-precision measurement date, the quan-
tifying experimental uncertainties are vital for results interpreting and analyzing process. Whereas creating an arrangement which has the least influence on the experiment is the first barrier. So the environmental specifications such as pressure, temperature, humidity and others are described below, the standards applies to apparatus designed to be safe under the succeeding circumstances:

a) Indoor use;

b) Temperature is from 5°C to 40°C;

c) Main supply voltage vacillations up to ±0.1 or ±10% of the nominal voltage;

d) Applicable pollution amount of the intended atmosphere (pollution degree is at most 2 in most cases);

e) Max relative humidity 80% for temperatures up to 30°C declining to 50% at 40°C;

f) Altitude up 2 Km;

1) Indoor laser measurement detection procedures:

Since the unguarded users can be exposed by this distinct class of laser, for the evaluation of moderate-risk laser instrument of the indoor experiment, the following step-by-step process is suggested:

- Step 1: The applicable AEL must be determined considering the max duration of exposure from the intended procedure. However, there is no such thing for experiment.
- Step 2: The dangerous beam path(s) must be determined
- Step 3: Because the reflection hazard fluctuates with the amount of beam focusing the surface nature, the extent of unsafe specular (mirror-like) reflection must be determined.
- Step 4: The extent of hazardous diffuse reflections (nominal danger zone) must be determined.
- Step 5: It must be determined whether any non-laser hazards exist or not in the system (broken mirrors, lenses).
2) Outdoor laser measurement detection procedures:
Defining the degree of several possibly hazardous circumstances is the main key for the evaluation of the total hazard of a certain laser device which can be achieved by the following step-by-step procedure:

- Phase 1: The applicable AEL must be determined considering the max duration of exposure from the intended procedure.
- Phase 2: The nominal hazardous zone of the laser must be estimated.
- Phase 3: The potential dangers from specular-surface reflections must be evaluated (i.e. mirrors in the wall and windows).
- Phase 4: If the laser is working in the range of 0.4 -1.4 µm (our experiment He-Ne laser is operating at $\lambda = 0.628 \, \mu$m), it must be determined whether nominal hazard region exist or not (Dangerous diffuse reflections).
- Phase 5: To determine both the level of horizontal and vertical range control, the stability of the laser platform must be evaluated.

4.2. Risk evaluation
4.2.1. Threats and Hazards

The personnel can control threats that can happen when doing the following: operating, disposal of laser equipment, installing or maintenance, basically by the defining the risks that they may encounter. So any physical situation, biological or even chemical, which might create damage (similarly known as personal injury) is called a hazard. Moreover, hazards not only cause only harm but also economical loss (i.e. damaged instruments and equipment).

Furthermore, the risk is composed of two things: the possibility of the harm that can happen and the generated harm severity. Unfortunately, it is impossible to completely eliminate the risk of injury (hazard). Still, lowering the risk during the experimentation to the minimum levels is mandatory to obtain at least partial safety.

Whereas, the application and the circumstances of use controls and define the acceptable level of risks (for example, minimum levels are defined by relating the
risk related to the experiment with consideration of analogous risks in other different environments).

The laser product class which is based on the max radiation intensity controls the accessibility for the individuals. Where the radiation hazards and threats are expressed and related to the laser product class. The table below review the control measures for certain applications.

When working with lasers, all personnel must be concerned with threats that a laser may cause, and must be trained to deal with such threats. Furthermore, when dealing with laser protection in general, there is a distinction between laser-non-specific and laser-specific risks, some of non-specific laser threats are:

- Risk of fire.
- Ozone formation by laser emission.
- Electrical threats (high voltage when working with high energy lasers).
- Explosion or implosion threats.
- Some laser systems operation may lead to X-rays exposure.
- When working with laser, sometimes may cause cooling medium exposure.

Generally, before purchasing the laser, we should assume that for a particular laser procedure there is an expectation to confront a risk related to it, which will create for the user the awareness of the safety conditions (where and how we can use and put the laser). The reduction of risk to reasonable degrees is a refined procedure. The table below shows the European classification of the laser products as well as their properties.

<table>
<thead>
<tr>
<th>IEC 60825</th>
<th>CDR H</th>
<th>ANSI Z136.1</th>
<th>Typical AEL for CW Lasers</th>
<th>Safety Aspects</th>
<th>Protective control measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 1</td>
<td>I</td>
<td>1 - 1</td>
<td>40 μW for blue</td>
<td>Safe.</td>
<td>Under conditions of normal procedure, no protective control measures are necessary</td>
</tr>
</tbody>
</table>
(under circumstances of maintenance or service it is another case).
For on-site servicing of embedded laser products, particular precautions are required.

<table>
<thead>
<tr>
<th>Class</th>
<th>I</th>
<th>M</th>
<th>Description</th>
<th>Safety Precautions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1M</td>
<td>Iia</td>
<td>-1M</td>
<td>Same as Class 1</td>
<td>Safe provided optical means are not used</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Avoid direct viewing of the laser over magnifying optical tools (microscopes, telescopes, binoculars and magnifying lenses) except when they have satisfactory degrees of protection. Avoid the usage of any external optical tools that increase the focus of the beam (beam divergence reduction)</td>
</tr>
<tr>
<td>2</td>
<td>II</td>
<td>2</td>
<td>1 mW</td>
<td>Avoid direct observation into the beam, and more importantly do not point it to other people or a crowd.</td>
</tr>
<tr>
<td>2M</td>
<td>IIIA</td>
<td>-2M</td>
<td>Same as Class 2</td>
<td>Avoid direct observation into the beam. More importantly do not point it to other people or a crowd.</td>
</tr>
<tr>
<td>Class</td>
<td>IIIb A</td>
<td>3a – 3R</td>
<td>5 times the limit of Class 1 in UV and IR, and 5 times the limit for Class 2 in visible, i.e. 5mW</td>
<td>Not safe, low risk</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>---------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>Class</td>
<td>IV</td>
<td>3B</td>
<td>500 mW</td>
<td>Hazardous, however observing of</td>
</tr>
</tbody>
</table>

The beam must always be ended at an appropriate non-specular plane (non-mirror like).

Avoid direct viewing of the laser over magnifying optical tools (microscopes, telescopes, binoculars and magnifying lenses) except when the have satisfactory degrees of protection.

Avoid the usage of any external optical tools that increase the focus of the beam (beam divergence reduction).
Finish Table 3

| Class | IV | 4 | No limit | Hazardous, however observing of diffuse reflection is dangerous, risk of fire. |

The essential stages involved in the risk evaluations are:

**4.2.2. Risk evaluation: phase 1 – possible harmful circumstances identification**

The consideration of every expected harmful circumstances that may happen when using the laser instruments, maintenance or any other activity related to the experiment or even expected failure is the risk evaluation principle step. By taking into consideration the activities systematically or arbitrarily, the list of “what may could go wrong?” can be obtained.

When ranking the potentially harmful circumstances, the personnel should concentrate on the following major topics:

1) **The laser atmosphere**

The laser atmosphere contains:

- The working area situation from the user perspective, or by other words, is it, bright or dark, dirty or dirt-free, is the job being performed simple or complex and is it ample or messy.
- The laser instrument site, which means the outer and the inner of a construction inside an enclosed and devoted laser operational region.
- The access degree: public access areas, open area where public access is denied, privileged area within the public access is denied.
- The working area situation from the instrument perspective, in other words; the consequence on instruments of humidity, dust, temperature and vibration …etc.

2) The involved threats
By taking into account the whole hazardous possibilities, the circumstances at which they could occur, the laser instrument type (its class, the conditions under which harmful injury may happen, and the type of injury) and even the job being performed play an extreme importance. When dealing with lasers, the exposure of laser is the most obvious threat which may lead to skin burn or losing eyesight, yet it is not the only one that may lead to other threats.

3) The personnel at risk
The service personnel, trained and qualified workers, visitors, people who may not understand fully the warning signs or the hazards that they may encounter (children, incapacitated, old people) are all classified as the people at risk.

4.2.2. Risk evaluation: phase 2 – evaluating risk for possibly harmful circumstances

The probability of injury and injury severity are the two features that incite the risk, by which they can be taken distinctly for each element on the possibly injurious circumstances. It can be difficult to quantify those factors. However, it happen frequently that after finishing the first phase of the risk evaluation procedure become very clear that an improper threat is happening and to reduce it, the steps must be done. To do that, it is really important to put evaluation on the conditions and threats that create dangerous degrees of exposure on the top of priorities.

The risk evaluation is described more properly in the following manner:

1) Risk seriousness
Risk seriousness (injury severity) is split to three groups:

- Minor: first aid is required. Nevertheless it is followed by a fast recovery.
- **Reasonable**: more serious outcome than the minor one, furthermore a medical treatment is desired and take longer time for recovery.

- **Major**: urgent medical treatment is required because of the critical injury, where sometimes it lead to death.

2) **Periodicity**

Concerning to the exposure time, the exposure to the hazard periodicity, there are three categories that periodicity of injury that could happen: - unlikely (occurrence is very improbable) – probable (occurrence is occasionally) – likely (frequently occurring).

3) **Resulting threat**

The decision for it to be appropriate or not is done by learning the consequential threat:

a) **Wavelength of the laser**

- The great threat comes from the ultraviolet emission, which can cause cancer if the skin is repeatedly exposed by the UV radiation.

- There is no such thing called “eye-safe” band, which means for any wavelength, when a laser has enough power there is a possibility for eye injury.

- The cornea shallow loss can heal, nevertheless, a deeper injuries into the cornea will not recover.

b) **Laser emission exposure period**

Because of the movement speed when feeling intense light, heat or pain, the period of exposure may be incomplete.

c) **Skin or eyes**

- Small-region burns are less serious than large-region burns, whatever the exposure degree.

- Very high lasers can lead to death or at least permanent costs.

- It is obvious to say that skin injuries are less serious than eyes injuries.
4.2.4. Risk evaluation: phase 3 – control measures selection

After control measures for reducing the threat is determined, if necessary, the process of the risk expectation (described below) must be repeated until the risk goes to the suitable level.

When the degree of the risk is undesirable the control measures must be presented to reduce it to the desired level.

The attention to the measures for decreasing the threat that can be made by the laser must be given first before selecting engineering controls and suitable controls.

When administrative controls cannot provide appropriate degree of safety, at least the personal protective instruments must be used.

4.3. Technical means to prevent electric shock and require basic safety.

The international standard IEC 61010-1 and 61140 is a necessary safety references for the personnel working in laser measurement experiments in laboratory rooms to prevent them from getting electrical shocks. Available conductive parts should not be hazardous-live-parts under:

- Normal circumstances.
- Single-fault circumstances.

For the instructed and skilled personnel and ordinary people, the accessibility rules may vary (also applied for different products and situations).

4.3.1. Primary protection means

To prevent accessible parts from becoming dangerous-live-parts, the following means should be followed:

a) Basic insulation

Based on the IEC 61010-1-2010, creepage distances plus clearances and solid insulation which make the basic isolation between hazardous-live-parts and the accessible parts should agree with the requirements.

b) Protective barriers or enclosure

Enclosures or protective should meet the requirements of 8.1 of IEC61010-1-2010 if they are going to deliver security by insulation.
The protective barriers should meet the requirements of 6.7 of IEC61010-1-2010 and basic safety applicable requirements if they are going to afford safety by clearances, restraining access and creepage distances between hazardous-live-parts and accessible parts.

c) Impedance

The following requirements should be satisfied for an impedance used as primary protection means:

- The voltage and the current should be limited by the impedance up to the appropriate degree.
- Impedance dissipation power quantity and the max operational voltage are the ones responsible of the impedance rate.

The international standard IEC 61010-1-2010 acceptable arrangement of protective means against electric shock is shown in figure 53.

![Figure 53 - Acceptable arrangement of protective means against electric shock](image-url)
CONCLUSION

In this project, two techniques for detecting the amount of orbital angular momentum in an optical vortex have been used, the first one is by using a variable rotational shearing interferometer which uses two prisms, one fixed and the other is rotated, the detection of topological charges by using this technique was simple and easy, where the generated interference pattern contains a double fork shape. The prongs of the fork are equal to the number of the topological charge plus one. However, this technique is only preferred when using optical vortex where the number of its topological charge corresponds to an even value. The second technique is by utilizing the reversal shearing interferometer which is based on the previous one where the rotated prism is replaced by a reflecting mirror. The resulting interfere pattern contains a fork shape that corresponds to the interference between a plane wavefront and an optical vortex. The topological charge of this vortex is the double of the topological charge of the vortex used in the technique, which opens the door for obtaining an optical vortices with integer charges by only using fractional vortices. The advantages of these techniques are: the simplicity, the flexibility and the low cost.

In the end, the detection of the topological charge of an optical vortex using rotational or reversal shearing interferometry was successful and a new and unique results have been achieved.

As for the future work, it will be focusing on the use of these two shearing interferometers with optical vortices that have a high topological charge number or fractional topological charges.


35. “HUYGENS' PRINCIPLE, Encyclopædia Britannica Online. Encyclopædia Britannica Inc.,”


APPENDIX A: MATLAB CODE

The full Matlab code used to generate the fork shapes is presented below:

```matlab
theta=zeros(600,400,'uint8');
for k=-5:5
    for i=-299:300
        for j=-199:200
            theta(i+300,j+200)=155*(1+(cos((k*atan(j/i))+(2*pi*i*(32*10^-6)*200))));
        end
    end
theta1=transpose(theta);
imshow(theta1)
imsave
end
```